SECTION 6.1: POWER SERIES (DAY 2)

(1) State the center of each power series below and find its **radius of convergence**, R and **interval of convergence**.

Use ratio test

(a) $\sum_{k=1}^{\infty} \frac{(x-2)^n}{\sqrt[3]{n}}$

$$\lim_{n\to\infty} \left| \frac{(x-2)^{n+1}}{\sqrt[3]{n+1}} \cdot \frac{\sqrt[3]{n}}{(x-2)^n} \right| = \lim_{n\to\infty} |x-2| \sqrt[3]{\frac{n}{n+1}} = |x-2|.$$

Want $|x-2|<1$, $\leftarrow R=1$

So $-1 < x-2 < 1$ or $1 < x < 3$

Check $x=3$: $\sum_{k=1}^{\infty} \frac{1}{\sqrt[3]{n}}$, divergent p-series

Check $x=1$: $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$. AST, $b_n = \frac{1}{n^{1/3}}$

$b_{n+1} = \frac{1}{\sqrt[3]{n+1}} < \frac{1}{\sqrt[3]{n}} = b_n$, $\lim_{n\to\infty} \frac{1}{\sqrt[3]{n}} = 0$. So

Series converges.

Answer $R=1$, I.o.C: $[1, 3)$

(b) $\sum_{k=1}^{\infty} \frac{(2x)^n}{5^n} = \sum \frac{2^n x^n}{5^n} =$

$= \sum \left(\frac{2}{5}\right)^n x^n$

$$\lim_{n\to\infty} \left| \frac{(2/5)^{n+1} x^{n+1}}{(2/5)^n x^n} \right| = \lim_{n\to\infty} |x| \cdot \frac{2}{5} = \frac{2}{5}|x|.$$

We want $\frac{2}{5}|x|<1$ or $|x| < \frac{5}{2}$ or $-\frac{5}{2} < x < \frac{5}{2}$.

$\uparrow R = 5/2$

Check: $x = 5/2$ $\quad \sum \left(\frac{2}{5}\right)^n \left(\frac{5}{2}\right)^n = \sum 1$ diverges.

check: $x = -5/2$ $\quad \sum \left(\frac{2}{5}\right)^n \left(\frac{-5}{2}\right)^n = \sum (-1)^n$ diverges.

Answer: $R = \frac{5}{2}$; I.o.C. $\left(-\frac{5}{2}, \frac{5}{2}\right)$

(c) $\sum_{k=1}^{\infty} \frac{(x-1)^n}{n!}$

$$\lim_{n\to\infty} \left| \frac{(x-1)^{n+1}}{(n+1)!} \cdot \frac{n!}{(x-1)^n} \right| = \lim_{n\to\infty} \frac{|x-1|}{n+1} = 0. \leftarrow \text{always } < 1.$$

Conclusion $R = \infty$, I.o.C is $(-\infty, \infty)$

(2) If you view the power series below as a **geometric series** what can you immediate conclude about (i) its radius and interval of converges and (ii) its sum (where it converges).

$$\sum_{k=0}^{\infty} x^n$$

From geometric series, we know $\sum_{k=0}^{\infty} x^n$ converges if $|x| < 1$

and diverges if $|x| \geq 1$.

If convergent, then $\sum_{k=0}^{\infty} x^n = \frac{1}{1-x} = f(x)$! !

(3) Use the formula above to write each function below as a power series. Determine its radius and interval of convergence.

(a) $f(x) = \frac{1}{1-9x^2}$ $= \sum_{k=0}^{\infty} (9x^2)^n = \sum_{k=0}^{\infty} 9^n x^{2n}$

ratio test: $\lim_{n\to\infty} \left| \frac{9^{n+1} x^{2n+2}}{9^n x^{2n}} \right| = \lim_{n\to\infty} 9x^2 = 9x^2 < 1$ So $x^2 < \frac{1}{9}$

$-\frac{1}{3} < x < \frac{1}{3}$. Check $x = \frac{1}{3}$, $\sum 1$ divergent. Check $x = -\frac{1}{3}$, $\sum (-1)^n$ divergent

$R = \frac{1}{3}$, I.o.C is $\left(-\frac{1}{3}, \frac{1}{3}\right)$

(b) $f(x) = \frac{x}{1+x}$

$= x\left(\frac{1}{1-(-x)}\right) = x \cdot \sum_{n=0}^{\infty} (-x)^n = x \sum_{n=0}^{\infty} (-1)^n x^n = \sum_{n=0}^{\infty} (-1)^n x^{n+1}$

ratio test: $\lim_{n\to\infty} \left| \frac{(-1)^{n+1} x^{n+2}}{(-1)^n x^{n+1}} \right| = \lim_{n\to\infty} |x| = |x| < 1$ $R = 1$, $-1 < x < 1$

$x = 1$: $\sum (-1)^n$ diverges I.o.C. $(-1, 1)$

$x = -1$: $\sum 1^n$ diverges