Section 6.3: Taylor and Maclaurin Series (Day 2)

(1) Recall from previous day:

If $f(x)$ has derivatives of all orders at $x = a$, then the **Taylor series** for $f(x)$ at $x = a$ is

(2) Recall the Taylor series for $y = \ln(x)$ at $x = 1$ is:

(3) Find the Taylor series for each function $f(x)$ at the given center $x = a$. (If you want to be ambitious, find their intervals of convergence!)

(a) $f(x) = e^{x/2}$ at $x = 0$

(b) $f(x) = \cos(x)$ at $a = \pi/2$

(4) Definition: The n-th Taylor *polynomial* of $f(x)$ centered at $x = a$ is:

$$p_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n.$$

(a) Find the first four Taylor polynomials, $p_0(x), p_1(x), p_2(x), p_3(x)$, for $f(x) = e^{x/2}$.

(b) Graph at least $f(x)$, $p_0(x)$ and $p_1(x)$ on the same set of axes. (If you want to be ambitious, graph $p_2(x)$ and $p_3(x)$ too.) Where have you seen $p_1(x)$ before?