Section 7.1: Parametric Equations

(1) Sketch the parametric equations below. Give the orientation of the curve.

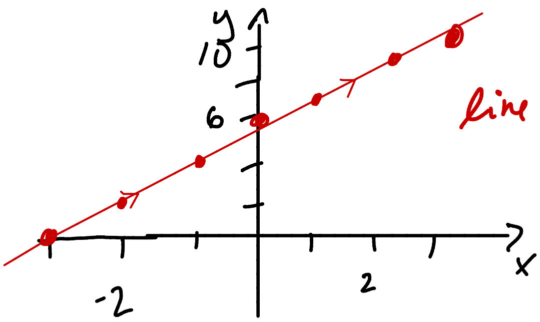
(a) $x(t) = t - 1,\ y(t) = 2t + 4$

t	-2	-1	0	1	2	3
x	-3	-2	-1	0	1	2
y	0	2	4	6	8	10

$t = x + 1$

$y = 2t + 4 = 2(x+1) + 4$

$y = 2x + 6$



line w/ direction

Note: Need to pay attention to see the direction after eliminating the parameter!

(b) $x(t) = \cos(t),\ y(t) = \sin(t)$

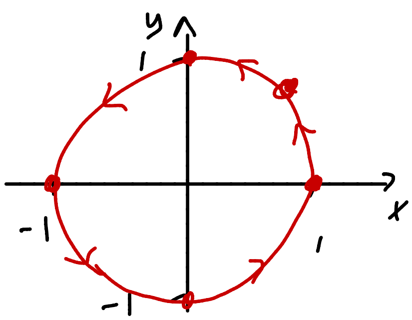
t	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
x	1	$\sqrt{2}/2$	0	-1	0	1
y	0	$\sqrt{2}/2$	1	0	-1	0

use:

$\sin^2 t + \cos^2 t = 1$

$x^2 + y^2 = 1$

circle.



(c) $x(t) = t^3,\ y(t) = 2t + 1$

t	-2	-1	0	1	2	3
x	-8	-1	0	1	8	27
y	-3	-1	1	3	5	7

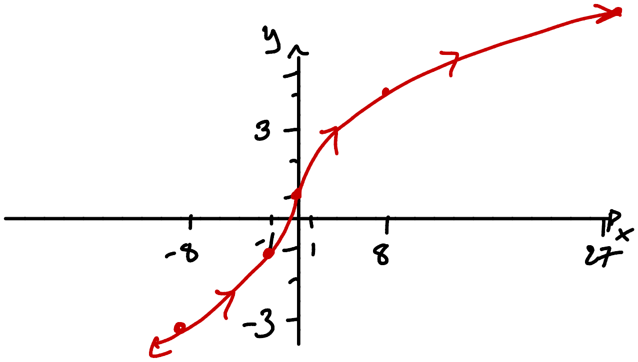
$y = 2t + 1$

So $t = \frac{y-1}{2}$

So $x = \left(\frac{y-1}{2}\right)^3$ or

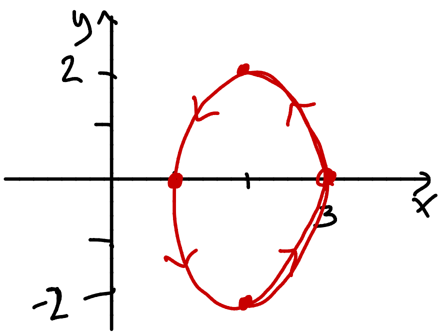
$x(y) = \frac{1}{8}(y-1)^3$

(cubic, shifted up 1 unit)



(d) $x(t) = 2 + \cos(t),\ y(t) = 2\sin(t)$

t	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	
x	3	2	1	2	0	
y	0	2	0	-2	0	



Again use $\cos^2 t + \sin^2 t = 1$

And,

$\cos(t) = x - 2$

$\sin(t) = \frac{y}{2}$

So $(x-2)^2 + \frac{y^2}{4} = 1$ ← an ellipse.

(2) For each problem above, eliminate the parameter.

(3) Find two different ways to parametrize $y = x^2$.

$x = t$, $y = t^2$ // $x = 2t$, $y = 4t^2$ // $x = -t$, $y = t^2$

What is the impact of different parametrizations?

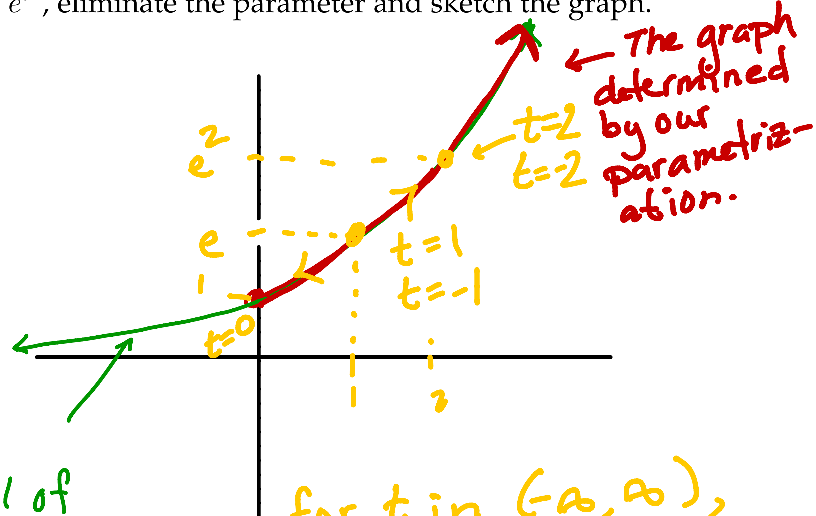
(4) For the parametric equations $x(t) = t^2,\ y(t) = e^{t^2}$, eliminate the parameter and sketch the graph. State the domain.

$x = t^2$, $y = e^x$

↑

but this parametrization forces $x \geq 0$.

So we do not graph all of $y = e^x$.



all of $y = e^x$

for t in $(-\infty, \infty)$, the graph is followed down to $y=1$ then back up again.

(5) Use technology to sketch the parametric equations below.

(a) $x(t) = 1 - \sin(t),\ y(t) = 1 - \cos(t)$

(b) $x(t) = 3\cos(t) + \cos(3t),\ y(t) = 3\sin(t) - \sin(3t)$