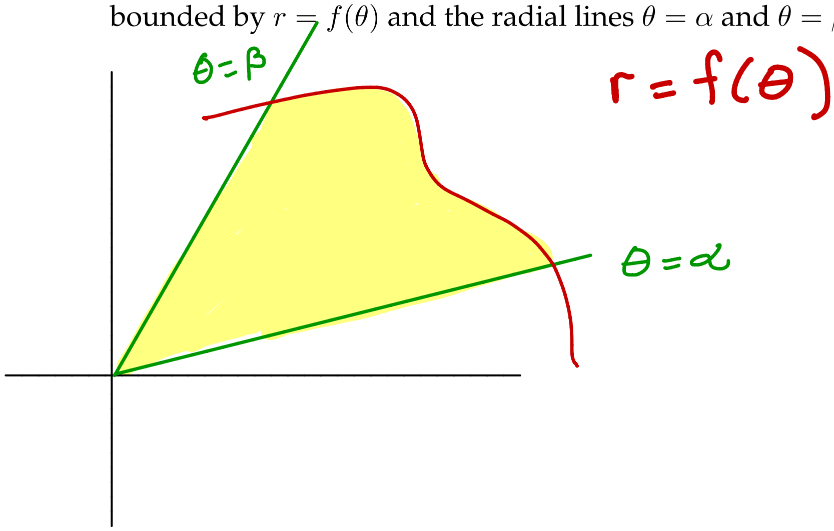
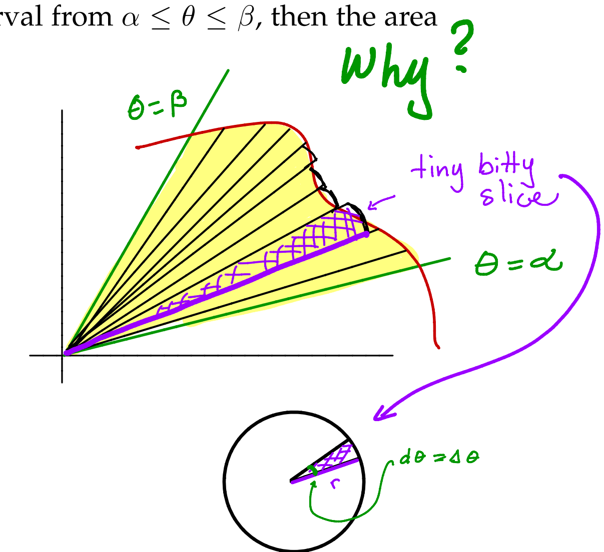
(1) Suppose $r = f(\theta)$ is a continuous and nonnegative on the interval from $\alpha \leq \theta \leq \beta$, then the area bounded by $r = f(\theta)$ and the radial lines $\theta = \alpha$ and $\theta = \beta$ is



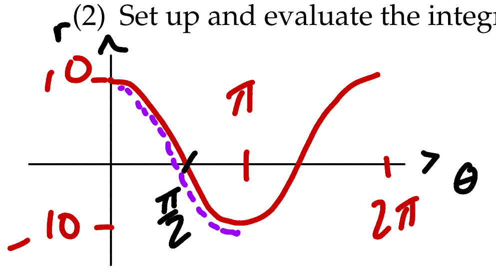


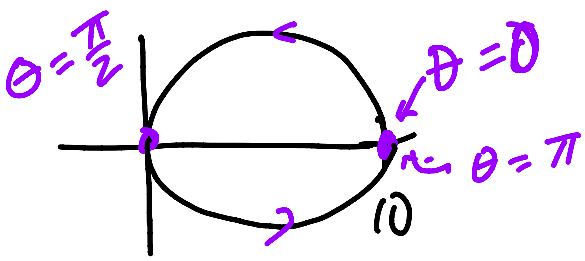
$$A = \int_\alpha^\beta \frac{1}{2}\left[f(\theta)\right]^2 d\theta$$

$$\frac{\boxed{\text{area sector}}}{\text{area circle}} = \frac{\Delta\theta}{2\pi} \quad \text{or} \quad \boxed{\text{area sector}} = \frac{(\text{area circle})}{2\pi}\, d\theta$$

$$\boxed{\text{area sector}} = \frac{\pi r^2}{2\pi}\, d\theta = \frac{1}{2} r^2 d\theta$$

(2) Set up and evaluate the integral to find the area enclosed by the polar curve $r = 10\cos(\theta)$.





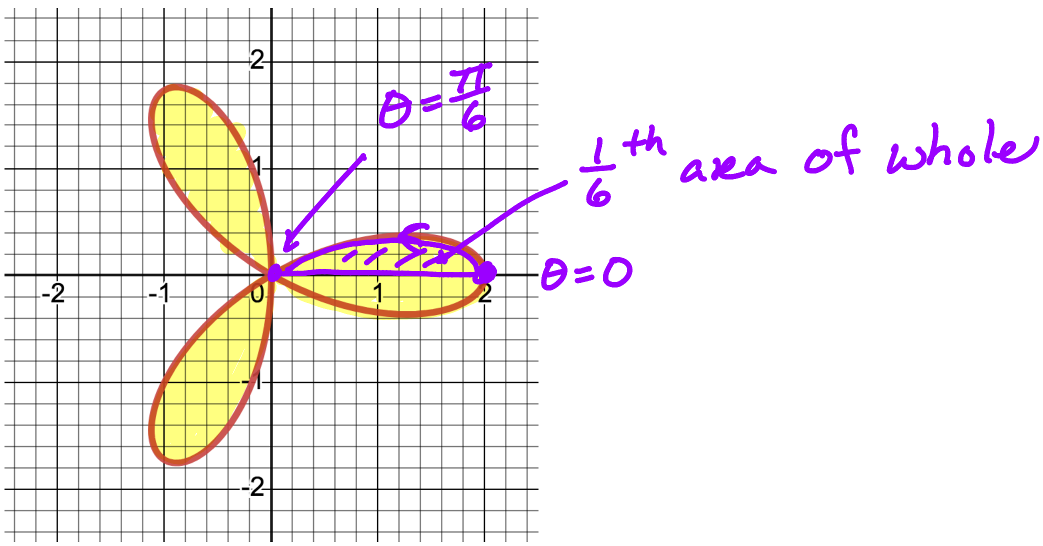
$$A = \int_0^\pi \frac{1}{2}(10\cos\theta)^2 d\theta = 50\int_0^\pi \cos^2\theta\, d\theta$$

$$= 25\int_0^\pi (1 + \cos(2\theta))\, d\theta$$

$$= 25\left(\theta + \frac{1}{2}\sin(\theta)\right)\Big|_0^\pi$$

$$= 25((\pi + 0) - 0)) = 25\pi$$

(3) Let R be the region enclosed by the polar curve $r = 2\cos(3\theta)$. Shade the region R, then Set up and evaluate the integral to find the area of R.



$$A = 6 \cdot \int_0^{\pi/6} \frac{1}{2}\left(2\cos(3\theta)\right)^2 d\theta$$

$$= 12 \int_0^{\pi/6} \cos^2(3\theta)\, d\theta = 6 \int_0^{\pi/6} \left(1 + \cos(6\theta)\right) d\theta$$

$$= 6\left(\theta + \frac{1}{6}\sin(6\theta)\right)\Bigg|_0^{\pi/6}$$

$$= 6\left(\left(\frac{\pi}{6} + 0\right) - 0\right) = \pi$$