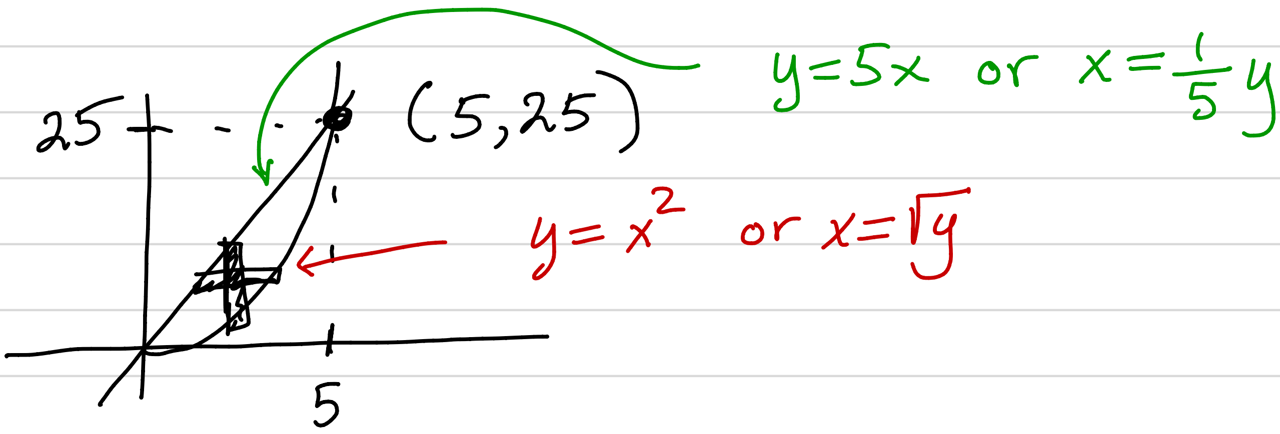
1. $V = \int_0^{\pi/4} \pi \sec^2 x \, dx = \tan(x) \Big|_0^{\frac{\pi}{4}} = \pi\left(\tan\left(\frac{\pi}{4}\right) - \tan(0)\right) = \pi(1-0) = \pi$

2.




a. $A = \int_0^5 (5x - x^2)\, dx$

b. $V = \pi \int_0^5 \left((5x)^2 - (x^2)^2\right) dx = \pi \int_0^5 (25x^2 - x^4)\, dx$

c. $V = 2\pi \int_0^{25} y\left(\sqrt{y} - \frac{1}{5}y\right) dy = 2\pi \int_0^{25} \left(y^{3/2} - \frac{1}{5}y^2\right) dy$

d. $V = \pi \int_0^{25} \left((\sqrt{y})^2 - \left(\frac{1}{5}y\right)^2\right) dy = \pi \int_0^{25} \left(y - \frac{y^2}{25}\right) dy$

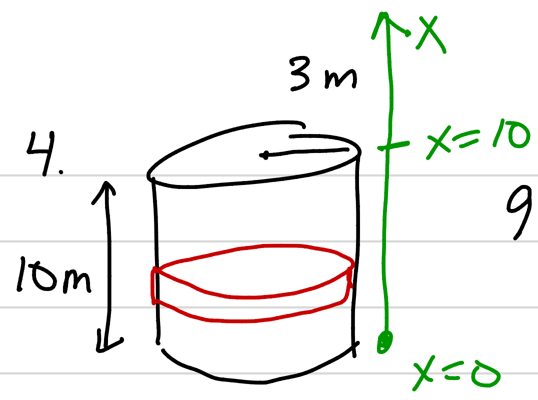
e. $V = 2\pi \int_0^5 x(5x - x^2)\, dx = 2\pi \int_0^5 (5x^2 - x^3)\, dx$

3. $F = Kx$ where $F = 10\,N$, $x = 0.2\,m$; So $10 = K(0.2)$. So $K = 50$.

$F = 50x$

$W = \int_0^{\frac{1}{2}} 50x \, dx = 25x^2 \Big]_0^{\frac{1}{2}} = \frac{25}{4}\, N\cdot m = \frac{25}{4}\, J$

4.




$9800 \ N/m^3$

$dV = \pi \cdot 3^2 \cdot dx = 9\pi dx \ m^3$

$dF = 9\pi(9800)dx \quad N$

$= 88200 \, dx \quad N$

$$W = \int_0^6 88200(10-x)\,dx = 88200\left(10x - x^2\right)\Big|_0^6 = 88200(60-36)$$

$$= 88200(24) \ N\cdot m = 2{,}116{,}800 \ J$$

5. $C: \ y = 6x^{3/2}$ on $[0,4]$ $\qquad y' = 9x^{1/2}$

a. $$AL = \int_0^4 \sqrt{1 + 81x}\, dx$$

b. $$SA = 2\pi \int_0^4 6x^{3/2} \sqrt{1+81x}\, dx$$

c. $x = \left(\frac{y}{6}\right)^{2/3}$ from $y=0$ to $y=48$ $\qquad x' = \frac{2}{3}\left(\frac{y}{6}\right)^{-1/3}$

$$SA = 2\pi \int_0^{48} \left(\frac{y}{6}\right)^{2/3} \sqrt{1 + \frac{4}{9}\left(\frac{y}{6}\right)^{-2/3}}\, dy$$

or

$$= 2\pi \int_0^4 x\sqrt{1+81x}\, dx$$

6. $\int \tan^3\theta \sec^4\theta \, d\theta = \int \tan^3\theta (1+\tan^2\theta)(\sec^2\theta \, d\theta)$

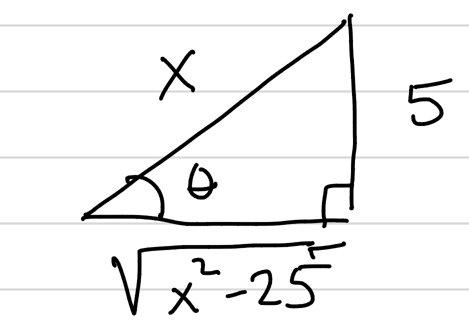
let $u = \tan\theta$
$du = \sec^2\theta \, d\theta$

$$= \int u^3(1+u^2)\,du = \int (u^3 + u^5)\,du$$

$$= \frac{1}{4}u^4 + \frac{1}{6}u^6 + C = \frac{1}{4}\tan^4\theta + \frac{1}{6}\tan^6\theta + C$$

7. $\int \frac{\sqrt{x^2-25}}{x}\,dx = \int \frac{5\tan\theta \cdot 5\sec\theta\tan\theta\,d\theta}{5\sec\theta}$

$x = 5\sec\theta$
$dx = 5\sec\theta\tan\theta\,d\theta$
$\sqrt{x^2-25} = 5\tan\theta$




$$= 5\int \tan^2\theta\,d\theta = 5\int(\sec^2\theta - 1)\,d\theta$$

$$= 5(\tan\theta - \theta) + C$$

$$= 5\left(\frac{5}{\sqrt{x^2-25}} - \arcsin\left(\frac{5}{x}\right)\right) + C$$

8. $\int x\sec^2(x)\,dx = x\tan x - \int \tan x\,dx$

$u = x \quad dv = \sec^2(x)\,dx$
$du = dx \quad v = \tan x$

$$= x\tan x - \int \frac{\sin x \, dx}{\cos x}$$

$$= x\tan x + \ln|\cos(x)| + C$$

9. $\int \frac{x^2+x+2}{x^3+x}\,dx = \int\left(\frac{2}{x} + \frac{-x+1}{x^2+1}\right)dx = \int\left(\frac{2}{x} - \frac{x}{x^2+1} + \frac{1}{x^2+1}\right)dx$

$\frac{x^2+x+2}{x^3+x} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$

$= 2\ln|x| - \frac{1}{2}\ln|x^2+1| + \arctan(x) + C$

$x^2+x+2 = A(x^2+1) + (Bx+C)(x)$

(equate coeff)

$2 = A$

$1 = C$

$1 = A+B$ so $B=-1$