# MIDTERM I PRACTICE PROBLEMS

Evaluate the integrals below. Your answers should be reasonably simplified.

1. Find the volume of the solid obtained by rotating the region bounded by the curves $y = \sec(x)$, $y = 0$, $x = 0$, and $x = \pi/4$ about the $x$- axis.

2. Consider the region $R$ bounded by the curves $y = x^2$ and $y = 5x$. Set up the integral(s) for each problem below.
   (a) The area of $R$.
   (b) The volume when $R$ is rotated about the $x$-axis using disks/washers.
   (c) The volume when $R$ is rotated about the $x$-axis using shells.
   (d) The volume when $R$ is rotated about the $y$-axis using disks/washers.
   (e) The volume when $R$ is rotated about the $y$-axis using shells.
   (f) Find the center of mass of $R$ assuming the density is constant.

3. Suppose it takes a force of 10 N to stretch a spring $0.2$ m from the equilibrium position. How much work is done to stretch the spring $0.5$ m from the equilibrium position?

4. Find the work done in pumping liquid out of the top of a cylindrical tank. The tank is 10 meters tall and is resting on its circular base which has a radius of 3 meters. Assume the liquid is 6 meters deep. The density of the liquid is $1000\,\text{kg/m}^3$. (Observe that the weight density or force density of the liquid is
$$(1000\,\text{kg/m}^3)(9.8\,\text{m/s}^2) = 9800\frac{(kg\cdot m)/s^2}{m^3} = 9800\frac{N}{m^3}.)$$

5. Let $C$ be the curve defined by $y = 6x^{3/2}$ between $x = 0$ and $x = 4$. Set up the integral for each problem below.
   (a) The length of the curve $C$.
   (b) The surface area if $C$ is rotated about the $x$-axis.
   (c) The surface area if $C$ is rotated about the $y$-axis.

6. $\displaystyle\int \tan^3\theta \sec^4\theta\, d\theta$

7. $\displaystyle\int \frac{\sqrt{x^2-25}}{x}\, dx$

8. $\displaystyle\int x\sec^2(x)\, dx$

9. $\displaystyle\int \frac{x^2+x+2}{x^3+x}\, dx$