MIDTERM II PRACTICE PROBLEMS

1. Indefinite Integrals

   (a) The directions for all such problems include the words *Use correct notation.* What does this mean in practice?

   (b) Compute and simplify the improper integral $\int_2^5 \frac{3x}{x^2-4}\,dx$ or show that it diverges. Use correct notation.

   (c) Check your **work** (not just your answer) against the solution. Did you use correct notation? Could anyone other than you follow and understand your work?

2. Sequences and Series: The Basics

   Let $a_n = \frac{n}{2n+1}$ for $n = 1, 2, 3, \cdots$

   (a) Write the first four terms in the **sequence** $a_1, a_2, a_3, \cdots$.

   (b) Does the **sequence** above converge? Show your work.

   (c) Write the **series** $\frac{1}{3} + \frac{2}{5} + \frac{3}{7} + \frac{4}{9} + \cdots$ using sum ($\sum$) notation.

   (d) Does the **series** above converge? Show that your answer is correct by stating the test you are using and applying that test.

   (e) Write S_1, S_2, and S_3, the first three terms in the sequence of partial sums of the **series**. Does the **sequence** $S_1, S_2, S_3, S_4, \cdots$ converge? Explain your answer.

3. Convervent and Divergent Series

   - Expect to use *different* tests. No midterm or final will be do-able by repeatedly using the same one or two tests over and over again.
   - Recall that a correct answer here is *not* "converge" or "diverge" since a coin can get that right 50% of the time. A correct answer is **a correct application of an appropriate test.** So the act of checking your answer should not be limited to the words "converge" or "diverge."
   - Check you **work.** Does it look anything like the solutions? Could anyone other than you follow and understand your work?

   Determine if the following series converge or diverge. Show your work.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{3\sqrt{n}+\pi}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^e}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{5}{n+\ln(n)}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{10^n}{(2n)!}$

   (e) $\displaystyle\sum_{n=0}^{\infty} \frac{n2^n}{5^n}$

   (f) $\displaystyle\sum_{n=2}^{\infty} \frac{\sin^3(n)}{n^2+1}$

4. Use the Integral Test to show that the series $\displaystyle\sum_{n=0}^{\infty} \frac{1}{1+n^2}$ converges.

5. Explain why the series $\displaystyle\sum_{n=2}^{\infty} \frac{(-2)^n}{7^n}$ converges and determine its sum. (HINT: Look carefully at the summation limits)

6. Find the interval of convergence for the power series $\displaystyle\sum_{n=0}^{\infty} \frac{1}{3^n}(x-1)^n$

7. Find a power series representation for $\frac{2x}{3+x}$

8. Write the Taylor series for $f(x) = e^{2x}$ centered at $x = 1$.