Integration Practice

Evaluate the integrals below. Your answers should be reasonably simplified.

1. $\int_0^{\pi/4} \sin^2(2\theta)\,dx = \frac{1}{2}\int_0^{\frac{\pi}{4}}(1-\cos(4\theta))\,d\theta = \frac{1}{2}\left(\theta - \frac{1}{4}\sin(4\theta)\right)\Big|_0^{\frac{\pi}{4}}$

$$= \frac{1}{2}\left(\left[\frac{\pi}{4}-\frac{1}{4}\sin(\pi)\right]-\left(0-\frac{1}{4}\sin(0)\right)\right)$$

$$= \frac{\pi}{8}$$

let $u=\sec\theta$;
$du=\sec\theta\tan\theta\,d\theta$

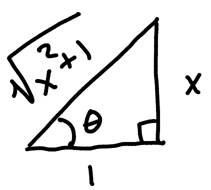
2. $\int x^3\sqrt{x^2+1}\,dx = \int \tan^3\theta\sec^3\theta\,d\theta = \int(\sec^2\theta-1)(\sec^2\theta)(\sec\theta\tan\theta\,d\theta)$

$x=\tan\theta$, $dx=\sec^2\theta\,d\theta$

$$= \int (u^2-1)(u^2)\,du = \int(u^4-u^2)\,du = \frac{1}{5}u^5-\frac{1}{3}u^3+C$$

$\sqrt{x^2+1} = \sec\theta$

$$= \frac{1}{5}\sec^5\theta-\frac{1}{3}\sec^3\theta+C = \frac{1}{5}(x^2+1)^{5/2}-\frac{1}{3}(x^2+1)^{3/2}+C$$



3. $\int_2^6 t^5\ln(t)\,dt = \frac{1}{6}t^6\ln(t)\Big]_2^6 - \frac{1}{6}\int_2^6 t^5\,dt = \left(6^5\ln(6)-\frac{2^5}{3}\ln(2)\right)-\frac{1}{6}\cdot\frac{1}{6}t^6\Big]_0^2$

let $u=\ln(t)$, $dv=t^5\,dt$

$du=\frac{1}{t}\,dt$, $v=\frac{1}{6}t^6$

$$= 6^5\ln(6)-\frac{2^5}{3}\ln(2)-\frac{1}{36}\left(2^6-0^6\right)$$

$36=4\cdot 9$

$$= 6^5\ln(6)-\frac{2^5}{3}\ln(2)-\frac{2^4}{9}$$

4. $\int_2^3 \frac{2x+3}{(x-1)(x+4)}\,dx = \int_2^3\left(\frac{1}{x-1}+\frac{1}{x+4}\right)dx = \ln|x-1|+\ln|x+4|\Big]_2^3$

$\frac{2x+3}{(x-1)(x+4)} = \frac{A}{x-1}+\frac{B}{x+4}$

So $2x+3 = A(x+4)+B(x-1)$

$x=-4$: $-5=-5B$; $B=1$

$x=1$: $5=5A$; $A=1$

$$= \left(\ln(2)+\ln(7)\right)-\left(\ln(1)+\ln(6)\right)$$

$$= \ln(2)+\ln(7)-\ln(6) = \ln\left(\frac{14}{6}\right) = \ln\left(\frac{7}{3}\right)$$

5. $\int \sec(x)\,dx = \ln|\sec x + \tan x| + C$

(trick: $\sec x = \sec x\left(\frac{\sec x + \tan(x)}{\sec x + \tan(x)}\right) = \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}$

6. $\int \frac{\ln(\ln(x))}{x\ln(x)}\,dx = \int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}\left(\ln(\ln(x))\right)^2 + C$

let $u = \ln(\ln(x))$

$du = \frac{1}{\ln(x)} \cdot \frac{1}{x}\,dx$

7. $\int \sin^6(x)\cos^3(x)\,dx = \int \sin^6(x)(1-\sin^2 x)\cos(x)\,dx = \int u^6(1-u^2)\,du$

pick $u = \sin(x)$
$du = \cos(x)\,dx$

$= \int (u^6 - u^8)\,du = \frac{1}{7}u^7 - \frac{1}{9}u^9 + C$

$= \frac{1}{7}(\sin x)^7 - \frac{1}{9}(\sin x)^9 + C$

8. $\int x\sin(7x)\,dx = -\frac{1}{7}x\cos(7x) + \frac{1}{7}\int \cos(7x)\,dx$

IBP

$u = x \qquad dv = \sin(7x)\,dx$

$du = dx \qquad v = -\frac{1}{7}\cos(7x)$

$= -\frac{1}{7}x\cos(7x) + \frac{1}{49}\sin(7x) + C$