SOLUTIONS



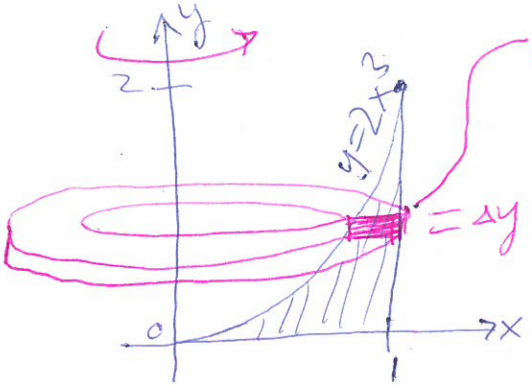
# Worksheet: Volumes by discs or washers.

Do these calculations with a group, if possible.

**A.** Sketch the region bounded by the given curves:

$$y = 2x^3, \quad x = 1, \quad y = 0.$$

Now sketch a typical slice and find the volume when the region is rotated around the $y$-axis.




$$\Delta V = \pi\left(1^2 - \left(\frac{y}{2}\right)^{2/3}\right)\Delta y$$

$$V = \int_0^2 \pi\left(1 - 2^{-2/3} y^{2/3}\right) dy$$

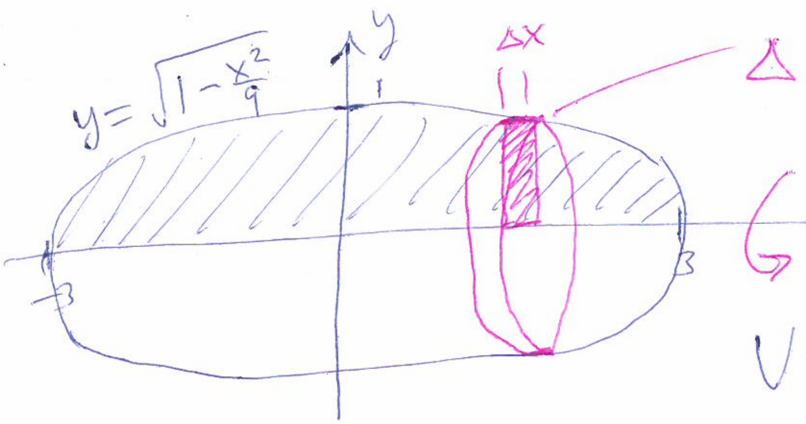
$$= \pi\left[y - 2^{-2/3}\frac{3}{5}y^{5/3}\right]_0^2$$

$$y = 2x^3 \Longleftrightarrow x = \left(\frac{y}{2}\right)^{1/3}$$

$$= \pi\left(2 - \frac{3}{5}\frac{2^{5/3}}{2^{2/3}}\right) = \pi\left(2 - \frac{3\cdot 2}{5}\right)$$

$$= \frac{4}{5}\pi$$

**B.** Sketch the ellipse $x^2 + 9y^2 = 9$. Rotate it around the $x$-axis, sketch a typical slice, and find the volume of the resulting rugby-ball-like ellipsoid.




$$\Delta V = \pi\left(\sqrt{1 - \frac{x^2}{9}}\right)^2 \Delta x$$

$$V = 2\int_0^3 \pi\left(1 - \frac{x^2}{9}\right) dx$$

$$= 2\pi\left[x - \frac{x^3}{27}\right]_0^3$$

$$= 2\pi[3 - 1] = 4\pi$$

$$x^2 + 9y^2 = 9$$

$$\frac{x^2}{9} + y^2 = 1$$

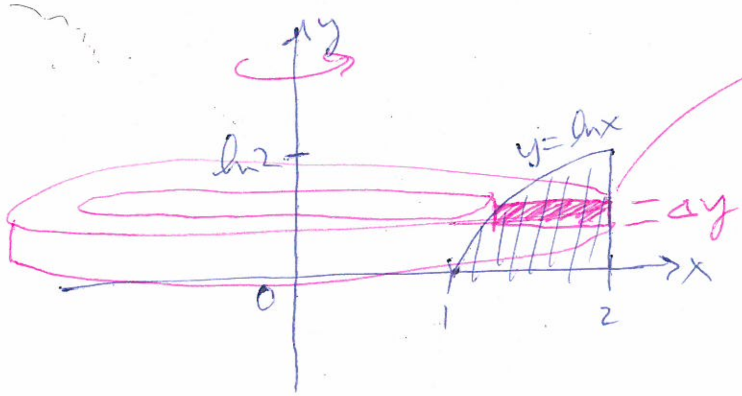
$$y = \pm\sqrt{1 - \frac{x^2}{9}}$$

**C.** Sketch the region bounded by the given curves:

$$y = \ln x, \quad x = 2, \quad y = 0.$$

Now sketch a typical slice and find the volume when the region is rotated around the $y$-axis.

$$\Delta V = \pi\left(2^2 - (e^y)^2\right)\Delta y$$




$y = \ln x$

$\Leftrightarrow \quad x = e^y$

$$V = \int_0^{\ln 2} \pi\left(4 - e^{2y}\right)dy$$

$$= \pi\left[4y - \frac{1}{2}e^{2y}\right]_0^{\ln 2}$$

$$= \pi\left[\left(4\ln 2 - \frac{1}{2}e^{2\ln 2}\right) - \left(0 - \frac{1}{2}\right)\right]$$

$$= \pi\left(4\ln 2 - \frac{1}{2}\cdot 2^2 + \frac{1}{2}\right) = \pi\left(4\ln 2 - \frac{3}{2}\right)$$

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