SOLUTIONS

Worksheet: Various trigonometric integrals

CORRECTED!

Compute these integrals with a group, if possible!

A. $\int \tan(4x)\,dx =$

$$\int \frac{\sin(4x)}{\cos(4x)}\,dx = \int \frac{-du/4}{u} = -\frac{1}{4}\int \frac{du}{u}$$

$u = \cos(4x)$

$-du/4 = \sin(4x)\,dx$

$$= -\frac{1}{4}\ln|\cos(4x)| + C$$

B. $\int \sec^2 x \tan^3 x\,dx =$

$$\int u^3\,du = \frac{1}{4}u^4 + C = \frac{1}{4}(\tan x)^4 + C$$

$u = \tan x$

$du = \sec^2 x\,dx$

C. $\int_0^\pi \sin(4x)\cos(3x)\,dx =$

$$\frac{1}{2}\int_0^\pi \sin(4x+3x) + \sin(4x-3x)\,dx$$

$$\left[\sin a\cos b = \frac{1}{2}(\sin(a+b) + \sin(a-b))\right]$$

$$= \frac{1}{2}\int_0^\pi \sin(7x) + \sin(x)\,dx = \frac{1}{2}\left[\frac{-\cos(7x)}{7} - \cos x\right]_0^\pi$$

$$= \frac{1}{2}\left[\left(\frac{1}{7}+1\right) - \left(\frac{-1}{7}-1\right)\right] = \frac{1}{2}\left[\frac{2}{7}+2\right]$$

$$= 8/7$$

CORRECTED

D. $\int \tan^4 t\,dt =$

$$= \int \tan^2 t(\sec^2 t - 1)\,dt = \int \tan^2 t \sec^2 t\,dt - \int \tan^2 t\,dt$$

$[\tan^2 t = \sec^2 t - 1]$

E. $\int \sec(2x)\,dx =$

$$= \int \sec(2x)\frac{\sec(2x)+\tan(2x)}{\sec(2x)+\tan(2x)}\,dx = \int \frac{\sec^2(2x)+\sec(2x)\tan(2x)}{\sec(2x)+\tan(2x)}\,dx$$

cont.

$$= \int \frac{du/2}{u} = \frac{1}{2}\ln|\sec(2x)+\tan(2x)| + C$$

$[u = \sec(2x)+\tan(2x)]$

D. cont

$$= \int \tan^2 t \sec^2 t \, dt - \int \tan^2 t \, dt$$

$\tan^2 t = \sec^2 t - 1$

$$= \int u^2 \, du - \int \sec^2 t - 1 \, dt$$

$(u = \tan t, \; du = \sec^2 t \, dt)$

$$= \frac{1}{3}(\tan t)^3 - \tan t + t + C$$