SOLUTIONS

Worksheet: Integrals of powers of sin and cos

Compute these integrals with a group, if possible!

A. $\displaystyle\int_{\pi/4}^{\pi/3} \cos^4 x \sin x \, dx = -\int_{\sqrt{2}/2}^{1/2} u^4\,du = \int_{1/2}^{\sqrt{2}/2} u^4\,du = \frac{1}{5}u^5\Big]_{1/2}^{\sqrt{2}/2}$

$u = \cos x$
$-du = \sin x\,dx$

$= \frac{1}{5}\left(2^{-5/2} - 2^{-5}\right)$

B. $\displaystyle\int \cos^3 x \sin^4 x \, dx = \int \cos^2 x \sin^4 x \cos x\,dx = \int (1-\sin^2 x)\sin^4 x \cos x\,dx$

$u = \sin x$
$du = \cos x\,dx$

$\cos^2 x = 1 - \sin^2 x$

$= \displaystyle\int (1-u^2)u^4\,du = \int u^4 - u^6\,du = \frac{1}{5}(\sin x)^5 - \frac{1}{7}(\sin x)^7 + C$

C. $\displaystyle\int \sin^2(4x)\, dx = \frac{1}{2}\int 1 - \cos(8x)\,dx$

$\sin^2\theta = \frac{1}{2}(1-\cos(2\theta))$

$= \frac{1}{2}\left(x - \frac{\sin(8x)}{8}\right) + C$

$= \frac{x}{2} - \frac{1}{16}\sin(8x) + C$

D. $\displaystyle\int e^{\sin x} \cos^3 x \, dx = \int e^{\sin x}(1-\sin^2 x)\cos x\,dx$

do IBP over!

$\cos^2 x = 1 - \sin^2 x$

$u = \sin x$
$du = \cos x\,dx$

$= \displaystyle\int e^u(1-u^2)\,du = \int e^u\,du - \int u^2 e^u\,du$

$= e^u - e^u(u^2 - 2u + 2) + C$

$= e^{\sin x}(-\sin^2 x + 2\sin x - 1) + C$

E. $\displaystyle\int \sin 2x \cos x \, dx =$

$= \displaystyle\int 2\sin x\cos x\cos x\,dx$

$\sin 2x = 2\sin x\cos x$

$= 2\displaystyle\int \cos^2 x \sin x\,dx$

$= 2\displaystyle\int u^2(-du) = -2\frac{u^3}{3} + C = -\frac{2}{3}(\cos x)^3 + C$

$u = \cos x$, $-du = \sin x\,dx$

on I:

$$\int u^2 e^u \, du = u^2 e^u - \int e^u \, 2u \, du$$

$w = u^2$ $z = e^u$

$dw = 2u\,du$ $dz = e^u\,du$

$$= u^2 e^u - 2\int u e^u \, du$$

$$= u^2 e^u - 2\left(u e^u - \int e^u \, du\right)$$

$s = u$ $t = e^u$

$ds = du$ $dt = e^u\,du$

$$= u^2 e^u - 2u e^u + 2\int e^u \, du$$

$$= u^2 e^u - 2u e^u + 2e^u + C$$

$$= e^u(u^2 - 2u + 2) + C$$