SOLUTIONS

Worksheet: Direction fields for differential equations

A differential equation (DE)

$$y' = f(x, y),$$

says that the *slope of the solution y' is determined by the location (x, y)*. Thus we can visualize the DE itself by drawing a *slope field* or *direction field*. An initial value problem (IVP) is a DE plus a point in the plane. The solution to an IVP is plotted by putting a dot at the initial value and then sketching a curve, the solution, through that dot that follows the direction field.

A. The direction field for $y' = x - y$ is shown below. Based on the direction field, sketch the solutions of the IVPs

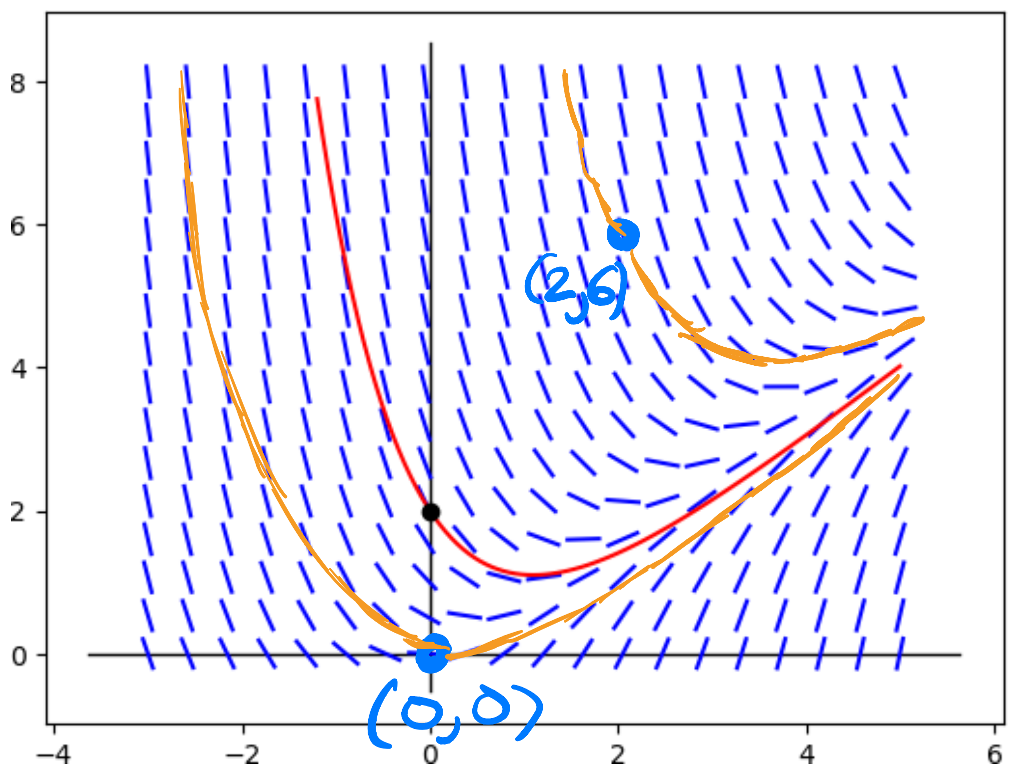
(i) $y' = x - y, \quad y(0) = 0$

(ii) $y' = x - y, \quad y(2) = 6$

Also, I claim $y(x) = x - 1 + 3e^{-x}$ is a solution to the IVP

$$y' = x - y, \quad y(0) = 2$$

Verify this. It is shown on the direction field already.



verify: $y' = 1 - 0 - 3e^{-x} = 1 - 3e^{-x}$ ✓

$x - y = x - (x - 1 + 3e^{-x}) = 0 + 1 - 3e^{-x}$

$2 = y(0) = 0 - 1 + 3e^{0} = -1 + 3 = 2$ ✓

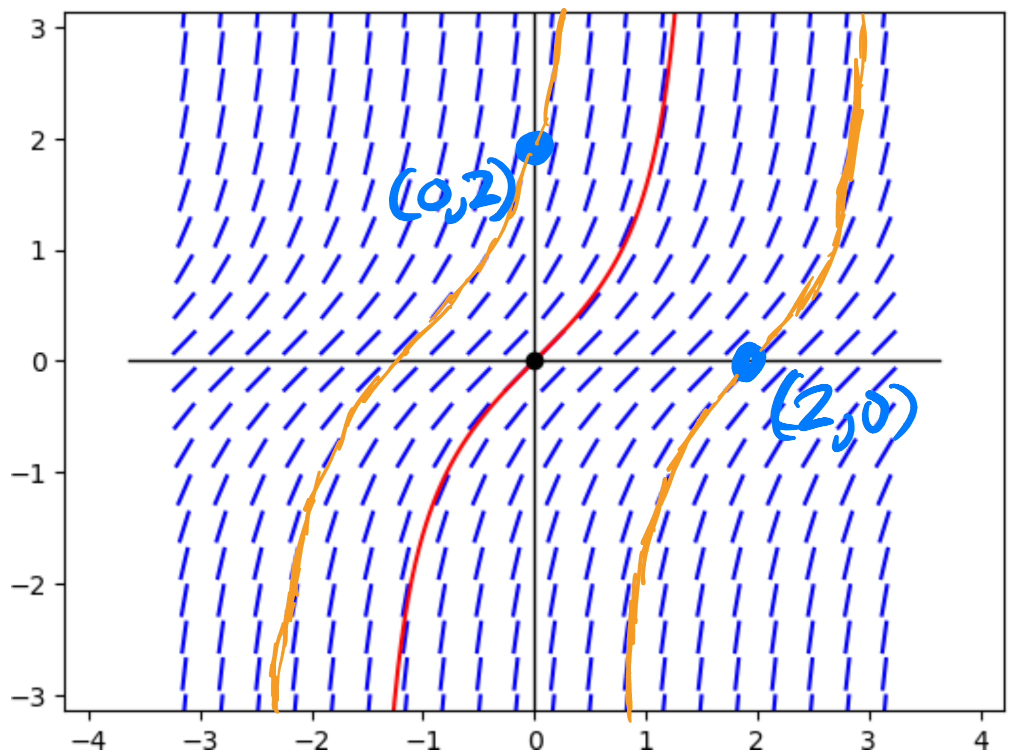
B. The direction field for $y' = 1 + y^2$ is shown below. Based on the direction field, sketch the solutions of the IVPs

(i) $y' = 1 + y^2, \quad y(0) = 2$
(ii) $y' = 1 + y^2, \quad y(2) = 0$

Also, I claim $y(x) = \tan(x)$ is a solution to the IVP

$$y' = 1 + y^2, \quad y(0) = 0$$

Verify this. It is shown on the direction field already.



verify: $y' = \sec^2 x$

$1 + y^2 = 1 + \tan^2 x = \sec^2 x$

$0 = y(0) = \tan 0 = 0$