SOLUTIONS



# Worksheet: Logarithms calculus practice (§2.7)

**A.** (like #307) Compute the derivative of $y = \log_7(\tan x)$

$= \frac{\ln(\tan x)}{\ln 7}$

$\frac{dy}{dx} = \frac{1}{\ln 7}\,\frac{1}{\tan x}\cdot \sec^2 x = \frac{\cos x}{(\ln 7)\sin x} = \frac{\cot x}{\ln 7}$

**B.** (#312) $\int_0^2 \frac{x\,dx}{x^2+1} =$

$\int_1^5 \frac{du/2}{u} = \frac{1}{2}[\ln u]_1^5$

$[u = x^2+1,\ du = 2x\,dx]$

$= \frac{\ln 5}{2}$

**C.** (#313) $\int_0^2 \frac{x^3\,dx}{x^2+1} =$

$\int_0^2 x - \frac{x}{x^2+1}\,dx = \left[\frac{x^2}{2}\right]_0^2 - \frac{\ln 5}{2}$

$\frac{x^3}{x^2+1} = \frac{x^3+x-x}{x^2+1} = \frac{x(x^2+1)}{x^2+1} - \frac{x}{x^2+1}$

$= 2 - \frac{\ln 5}{2}$

**D.** (like #314) $\int_2^e \frac{dx}{x(\ln x)^2} =$

$[u = \ln x,\ du = \frac{dx}{x}]$

$\int_{\ln 2}^1 \frac{du}{u^2} = \left[-u^{-1}\right]_{\ln 2}^1$

$= -\frac{1}{1} + \frac{1}{\ln 2} = \frac{1}{\ln 2} - 1$

**E.** (#317) $\int_0^{\pi/4} \tan x\,dx =$

$\int_0^{\pi/4} \frac{\sin x}{\cos x}\,dx = \int_1^{1/\sqrt{2}} \frac{-du}{u} = \int_{1/\sqrt{2}}^1 \frac{du}{u}$

$[u = \cos x,\ du = -\sin x\,dx]$

$= \ln 1 - \ln(1/\sqrt{2})$

$= \frac{1}{2}\ln 2$

**F.** (like #322) Compute the derivative of $y = x^{\sin x}$. (*Hint. Find the derivative of* $\ln y$.)

$\ln y = (\sin x)\ln x$

$\frac{1}{y}\frac{dy}{dx} = (\cos x)\ln x + (\sin x)\frac{1}{x}$

$\frac{dy}{dx} = \left(\cos x \cdot \ln x + \frac{\sin x}{x}\right) x^{\sin x}$