SOLUTIONS

Worksheet: Integrals of powers of sin and cos

Compute these integrals with a group, if possible!

A. $\int_{\pi/4}^{\pi/3} \cos^4 x \sin x \, dx = -\int_{\sqrt{2}/2}^{1/2} u^4 \, du = \int_{1/2}^{\sqrt{2}/2} u^4 \, du = \frac{1}{5} u^5 \Big]_{1/2}^{\sqrt{2}/2}$

$u = \cos x$
$-du = \sin x \, dx$

$= \frac{1}{5}\left(2^{-5/2} - 2^{-5}\right)$

B. $\int \cos^3 x \sin^4 x \, dx = \int \cos^2 x \sin^4 x \cos x \, dx = \int (1-\sin^2 x)\sin^4 x \cos x \, dx$

$\cos^2 x = 1 - \sin^2 x$

$u = \sin x$
$du = \cos x \, dx$

$= \int (1-u^2) u^4 \, du = \int u^4 - u^6 \, du = \frac{1}{5}(\sin x)^5 - \frac{1}{7}(\sin x)^7 + C$

C. $\int \sin^2(4x) \, dx = \frac{1}{2}\int 1 - \cos(8x) \, dx$

$\sin^2\theta = \frac{1}{2}(1-\cos(2\theta))$

$= \frac{1}{2}\left(x - \frac{\sin(8x)}{8}\right) + C$

$= \frac{x}{2} - \frac{1}{16}\sin(8x) + C$

D. $\int e^{\sin x} \cos^3 x \, dx = \int e^{\sin x}(1-\sin^2 x)\cos x \, dx$

$\cos^2 x = 1 - \sin^2 x$

do IBP over!

$u = \sin x$
$du = \cos x \, dx$

$= \int e^u (1-u^2) \, du = \int e^u \, du - \int u^2 e^u \, du$

$= e^u - e^u(u^2 - 2u + 2) + C$

$= e^{\sin x}(-\sin^2 x + 2\sin x - 1) + C$

E. $\int \sin 2x \cos x \, dx =$

$= \int 2 \sin x \cos x \cos x \, dx$

$\sin 2x = 2\sin x \cos x$

$= 2\int \cos^2 x \sin x \, dx$

$= 2\int u^2(-du) = -2\frac{u^3}{3} + C = -\frac{2}{3}(\cos x)^3 + C$

$u = \cos x, \ -du = \sin x \, dx$

on I:

$$\int u^2 e^u\,du = u^2 e^u - \int e^u\, 2u\,du$$

$w = u^2 \quad z = e^u$

$dw = 2u\,du \quad dz = e^u\,du$

$$= u^2 e^u - 2\int u e^u\,du$$

$$= u^2 e^u - 2\left(u e^u - \int e^u\,du\right)$$

$s = u \quad t = e^u$

$ds = du \quad dt = e^u\,du$

$$= u^2 e^u - 2u e^u + 2\int e^u\,du$$

$$= u^2 e^u - 2u e^u + 2e^u + C$$

$$= e^u(u^2 - 2u + 2) + C$$

SOLUTIONS

Worksheet: Various trigonometric integrals

CORRECTED!

Compute these integrals with a group, if possible!

A. $\int \tan(4x)\,dx = \int \frac{\sin(4x)}{\cos(4x)}\,dx = \int \frac{-du/4}{u} = -\frac{1}{4}\int \frac{du}{u}$

$u = \cos(4x)$
$-du/4 = \sin(4x)\,dx$

$= -\frac{1}{4}\ln|\cos(4x)| + C$

B. $\int \sec^2 x \tan^3 x\,dx = \int u^3\,du = \frac{1}{4}u^4 + C = \frac{1}{4}(\tan x)^4 + C$

$u = \tan x$
$du = \sec^2 x\,dx$

C. $\int_0^{\pi} \sin(4x)\cos(3x)\,dx = \frac{1}{2}\int_0^{\pi} \sin(4x+3x) + \sin(4x-3x)\,dx$

CORRECTED

$\left[\sin a \cos b = \frac{1}{2}(\sin(a+b) + \sin(a-b))\right]$

$= \frac{1}{2}\int_0^{\pi} \sin(7x) + \sin(x)\,dx = \frac{1}{2}\left[\frac{-\cos(7x)}{7} - \cos x\right]_0^{\pi}$

$= \frac{1}{2}\left[\left(\frac{1}{7}+1\right) - \left(\frac{-1}{7}-1\right)\right] = \frac{1}{2}\left[\frac{2}{7}+2\right]$

$= 8/7$

D. $\int \tan^4 t\,dt =$

$[\tan^2 t = \sec^2 t - 1]$

$= \int \tan^2 t(\sec^2 t - 1)\,dt = \int \tan^2 t \sec^2 t\,dt - \int \tan^2 t\,dt$

E. $\int \sec(2x)\,dx =$

cont.

$= \int \sec(2x)\frac{\sec(2x)+\tan(2x)}{\sec(2x)+\tan(2x)}\,dx = \int \frac{\sec^2(2x) + \sec(2x)\tan(2x)}{\sec(2x)+\tan(2x)}\,dx$

$= \int \frac{du/2}{u} = \frac{1}{2}\ln|\sec(2x) + \tan(2x)| + C$

$[u = \sec(2x) + \tan(2x)]$

D. cont

$$= \int \tan^2 t \sec^2 t \, dt - \int \tan^2 t \, dt$$

$\left[\tan^2 t = \sec^2 t - 1\right]$

$$= \int u^2 \, du - \int \sec^2 t - 1 \, dt$$

$\left(\begin{array}{l} u = \tan t \\ du = \sec^2 t \, dt \end{array}\right)$

$$= \frac{1}{3}(\tan t)^3 - \tan t + t + C$$