SOLUTIONS

1. Express the rational function as a sum of simpler rational functions. That is: **expand in partial fractions**.

(a) *like 3.4 #182* $\frac{2}{(x-1)(x-3)} = \frac{A}{x-1} + \frac{B}{x-3} = \boxed{\frac{-1}{x-1} + \frac{1}{x-3}}$

$0x + 2 = A(x-3) + B(x-1) = (A+B)x + (-3A-B)$

$\left.\begin{matrix} A+B=0 \\ -3A-B=2 \end{matrix}\right\}$ $-2A=2$, $A=-1, B=1$

(b) *3.4 #183* $\frac{x^2+1}{x(x+1)(x+2)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x+2} = \boxed{\frac{1}{2x} - \frac{2}{x+1} + \frac{5/2}{x+2}}$

$1x^2 + 0x + 1 = x^2+1 = A(x+1)(x+2) + Bx(x+2) + Cx(x+1)$

$= (A+B+C)x^2 + (3A+2B+C)x + 2A$

$\left.\begin{matrix} A+B+C=1 \\ 3A+2B+C=0 \\ 2A=1 \end{matrix}\right\} \Rightarrow A=\frac{1}{2}, B=-2, C=\frac{5}{2}$

(c) *3.4 #188* $\frac{1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1} = \boxed{\frac{1/2}{x-1} - \frac{\frac{1}{2}x+\frac{1}{2}}{x^2+1}}$

$0x^2 + 0x + 1 = 1 = A(x^2+1) + (Bx+C)(x-1)$

$\left.\begin{matrix} A+B=0 \\ -B+C=0 \\ A-C=1 \end{matrix}\right\} \Rightarrow A=\frac{1}{2}, B=-\frac{1}{2}, C=-\frac{1}{2}$

2. **Evaluate the integrals using partial fractions.**

(a) *3.4 #204* $\int \frac{2}{x^2-x-6}\,dx = \int \frac{-2/5}{x+2} + \frac{2/5}{x-3}\,dx$

$\frac{2}{(x+2)(x-3)} = \frac{A}{x+2} + \frac{B}{x-3}$

$0x+2 = A(x-3) + B(x+2)$

$\left.\begin{matrix} A+B=0 \\ -3A+2B=2 \end{matrix}\right\} A=-\frac{2}{5}, B=\frac{2}{5}$

$= \boxed{-\frac{2}{5}\ln|x+2| + \frac{2}{5}\ln|x-3| + C}$

$= \frac{2}{5}\ln\left|\frac{x-3}{x+2}\right| + C$

(b) *like 3.4 #211* $\displaystyle\int \frac{x+3}{(x^2+1)(x-4)}\,dx =$ $\displaystyle\int \frac{-7/17\,x - 11/17}{x^2+1} + \frac{7/17}{x-4}\,dx$

$$\frac{x+3}{(x^2+1)(x-4)} = \frac{Ax+B}{x^2+1} + \frac{C}{x-4}$$

$$0x^2 + 1x + 3 = (Ax+B)(x-4) + C(x^2+1)$$

$$\left.\begin{aligned} A + C &= 0 \\ -4A + B &= 1 \\ -4B + C &= 3 \end{aligned}\right\} A = \frac{-7}{17},\ B = \frac{-11}{17},\ C = \frac{7}{17}$$

$u = x^2+1$

$$= -\frac{7}{17}\int \frac{x}{x^2+1}\,dx - \frac{11}{17}\int \frac{dx}{x^2+1} + \frac{7}{17}\int \frac{dx}{x-4}$$

$$= \boxed{\frac{-7}{34}\ln(x^2+1) - \frac{11}{17}\arctan x + \frac{7}{17}\ln|x-4| + C}$$

(c) *like 3.4 #203* $\displaystyle\int_1^2 \frac{2-x}{x^2+x}\,dx =$ $\displaystyle\int_1^2 \frac{-3}{x+1} + \frac{2}{x}\,dx$

$$\frac{2-x}{(x+1)x} = \frac{A}{x+1} + \frac{B}{x}$$

$$2 - x = Ax + B(x+1)$$

$$\left.\begin{aligned} A + B &= -1 \\ B &= 2 \end{aligned}\right\} A = -3,\ B = 2$$

$$= -3\ln|x+1|\Big]_1^2 + 2\ln|x|\Big]_1^2$$

$$= -3\ln 3 + 3\ln 2 + 2\ln 2 - 2\overset{0}{\cancel{\ln 1}}$$

$$= -3\ln 3 + 5\ln 2 = 0.1699$$

(d) *3.4 #227; hint: start with a substitution* $\displaystyle\int \frac{1}{1+e^x}\,dx =$ $\displaystyle\int \frac{1}{1+u}\,\frac{du}{u}$

$$\boxed{\begin{aligned} u &= e^x \\ du &= e^x\,dx \\ \frac{du}{u} &= dx \end{aligned}}$$

$$\frac{1}{(1+u)u} = \frac{A}{1+u} + \frac{B}{u}$$

$$0u + 1 = Au + B(1+u)$$

$$\left.\begin{aligned} A + B &= 0 \\ B &= 1 \end{aligned}\right\} A = -1,\ B = 1$$

$$= \int \frac{-1}{1+u} + \frac{1}{u}\,du$$

$$= -\ln|1+u| + \ln|u| + C$$

$$= \boxed{-\ln|1+e^x| + x + C}$$