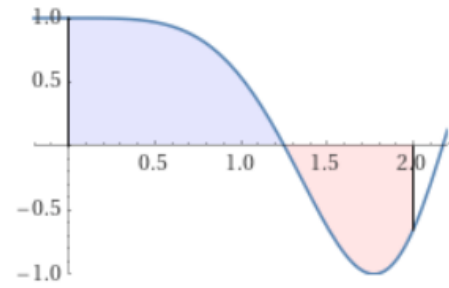


## SECTION 3.6: NUMERICAL INTEGRATION

We will try to estimate the definite integral

$$\int_0^2 \cos(x^2) dx$$

I do not know how to do it by hand exactly. (*Feel free to try?*) However, we can graph the function  $y = \cos(x^2)$ . Eyeballing the graph at right, the area above the axis is about 1 and the area below is about 1/2, so we expect a final integral of about 1/2.



1. Write down the Midpoint Rule  $M_4$  for this integral, with  $n = 4$  subintervals. (*What are the values of  $\Delta x$  and the points  $m_i$ ?*)
2. Use a calculator to evaluate  $M_4$ . Round your estimate to 4 decimal places.
3. Write down the Trapezoid Rule  $T_4$  for this integral, with  $n = 4$  subintervals. (*What are the values of  $\Delta x$  and the points  $x_i$ ?*)
4. Use a calculator to evaluate  $T_4$ . Round your estimate to 4 decimal places.

5. Write down Simpson's Rule  $S_4$  for this integral, with  $n = 4$  subintervals. (What are the values of  $\Delta x$  and the points  $x_i$ ?)

6. Use a calculator to evaluate  $S_4$ . Round your estimate to 4 decimal places.

7. In Matlab, the command

```
>> integral(@(x) cos(x.^2), 0, 2)
```

gives the 0.461461462433216 as an estimate. Using this number as the exact value of the integral, determine the *absolute* error for each of the three estimates  $M_4$ ,  $T_4$ ,  $S_4$ .