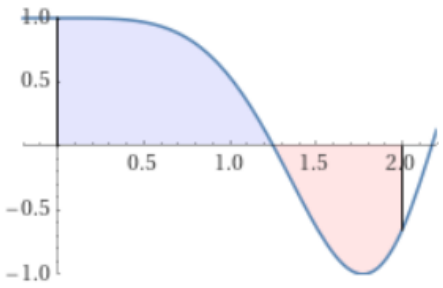
SOLUTIONS

SECTION 3.6: NUMERICAL INTEGRATION

We will try to estimate the definite integral

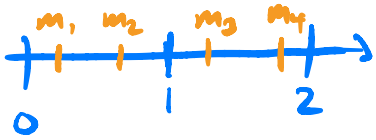
$$\int_0^2 \cos(x^2)\,dx \qquad f(x)=\cos(x^2)$$



I do not know how to do it by hand exactly. (*Feel free to try?*) However, we can graph the function $y = \cos(x^2)$. Eyeballing the graph at right, the area above the axis is about 1 and the area below is about 1/2, so we expect a final integral of about 1/2.

1. Write down the Midpoint Rule M_4 for this integral, with $n = 4$ subintervals. (*What are the values of* Δx *and the points* m_i*?*)

$\Delta x = (2-0)/4 = \frac{1}{2}$, $m_1 = 0.25$, $m_2 = 0.75$, $m_3 = 1.25$, $m_4 = 1.75$



$$M_4 = \Delta x\left(f(m_1)+f(m_2)+f(m_3)+f(m_4)\right)$$

2. Use a calculator to evaluate M_4. Round your estimate to 4 decimal places.

$$M_4 = 0.4277$$

3. Write down the Trapezoid Rule T_4 for this integral, with $n = 4$ subintervals. (*What are the values of* Δx *and the points* x_i*?*)

$\Delta x = \frac{1}{2}$, $x_0 = 0$, $x_1 = \frac{1}{2}$, $x_2 = 1$, $x_3 = 1.5$, $x_4 = 2$

$$T_4 = \frac{\Delta x}{2}\left(f(x_0)+2f(x_1)+2f(x_2)+2f(x_3)+f(x_4)\right)$$

4. Use a calculator to evaluate T_4. Round your estimate to 4 decimal places.

$$T_4 = 0.5271$$

5. Write down Simpson's Rule S_4 for this integral, with $n = 4$ subintervals. (*What are the values of* Δx *and the points* x_i*?*)

same as for trapezoid

$\Delta x = \frac{1}{2}$, $x_0 = 0$, $x_1 = \frac{1}{2}$, $x_2 = 1$, $x_3 = \frac{3}{2}$, $x_4 = 2$

$$S_4 = \frac{\Delta x}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\right)$$

6. Use a calculator to evaluate S_4. Round your estimate to 4 decimal places.

$S_4 = 0.4650$

7. In Matlab, the command

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>> integral(@(x) cos(x.^2),0,2)
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$= I$

gives the 0.461461462433216 as an estimate. Using this number as the exact value of the integral, determine the *absolute* error for each of the three estimates M_4, T_4, S_4.

$|M_4 - I| = 0.34$

$|T_4 - I| = 0.66$

} comparable

$|S_4 - I| = 0.0035$ } substantially more accurate for same amount of work