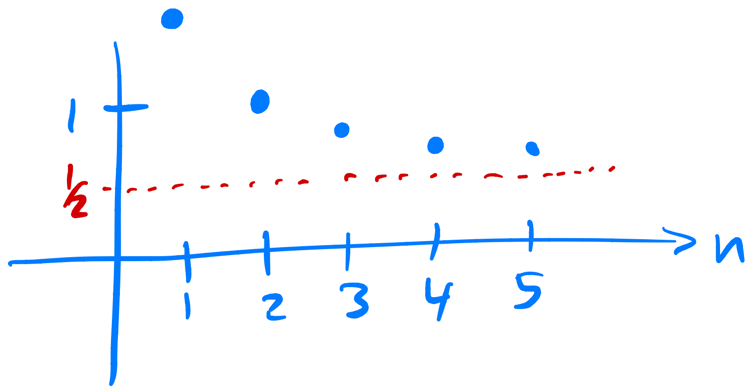
SOLUTIONS

1. To know by the end of section 5.1:
   (a) what an infinite sequence is
   (b) how to read and use sequence notation
   (c) what it means to "Find a formula for the nth term", and how to find it
   (d) what it means for a sequence to converge or diverge
   (e) different limit techniques for determining if a sequence converges or diverges, including L'Hopital's rule
   (f) what $n!$ means
   (g) terms for describing a sequence: bounded, monotone, increasing, decreasing

2. For each sequence below, write the first 5 terms and graph them.

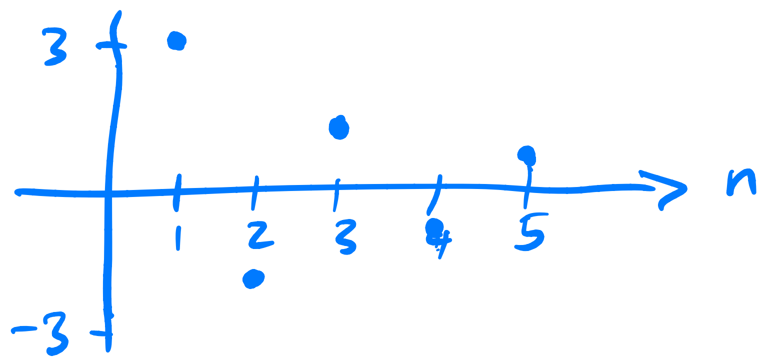
   (a) $\left\{\dfrac{n+2}{2n}\right\}_{n=1}^{\infty}$

   $\frac{3}{2}, \frac{4}{4}, \frac{5}{6}, \frac{6}{8}, \frac{7}{10}$



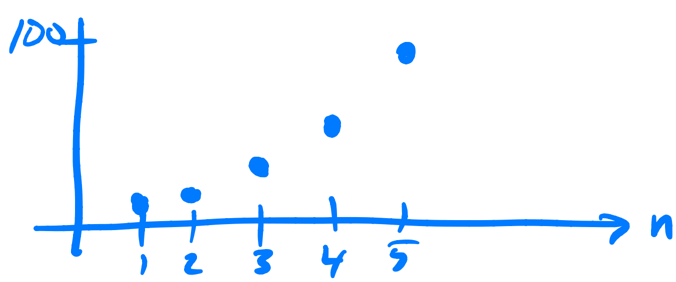
   (b) $a_n = 3\left(\dfrac{-1}{2}\right)^{n-1}$ for $n \geq 1$

   $3, -\frac{3}{2}, +\frac{3}{4}, -\frac{3}{8}, +\frac{3}{16}$



   (c) $a_1 = 5$ and $a_n = 2\,a_{n-1} + 1$

   $5, 11, 23, 47, 95$



3. Find a formula for 2 (c).

$$a_n = 6 \cdot 2^{n-1} - 1$$

$\left[6, 12, 24, 48, 96, \ldots = 6 \cdot 2^{n-1}\right]$

4. Definition: The symbol $n!$ or "n factorial" means

$$n! = n(n-1)(n-2)\cdots(2)(1)$$

and $0! = 1$

5. Find the limit of each of the following sequences or show that it diverges.

(a) $\left\{\pi + \frac{100}{n}\right\}$

$$\lim_{n\to\infty} \pi + \frac{100}{n} = \pi + 0 = \pi$$

(b) $a_n = \frac{3^n}{n!}$ $= \dfrac{3\cdot 3\cdot \cdots \cdot 3}{\underbrace{n\cdot(n-1)}\cdot\cdots 1}$ $\therefore \lim_{n\to\infty} \frac{3^n}{n!} = 0$

product of $\frac{3}{n}, \frac{3}{n-1}, \ldots$ is small

(c) $\left\{\frac{100n^2 + \sqrt{n}}{n - 3n^2}\right\}$

$$\lim_{n\to\infty} \frac{100n^2+\sqrt{n}}{n-3n^2} \frac{\cdot\frac{1}{n^2}}{\cdot\frac{1}{n^2}} = \lim_{n\to\infty} \frac{100 + \frac{1}{n^{3/2}}}{\frac{1}{n} - 3} = \frac{100+0}{0-3} = \frac{-100}{3}$$

(d) $\left\{\frac{n^2}{10^n}\right\}$

$$\lim_{n\to\infty} \frac{n^2}{10^n} \underset{\frac{\infty}{\infty}}{\overset{L'H}{=}} \lim_{n\to\infty} \frac{2n}{(\ln 10)10^n} \underset{\frac{\infty}{\infty}}{\overset{L'H}{=}} \lim_{n\to\infty} \frac{2}{(\ln 10)^2 10^n} = 0$$

(e) $a_n = \left(1 + \frac{1}{n}\right)^n$

$$\ln a_n = n \ln\left(1+\tfrac{1}{n}\right)$$

$$\lim_{n\to\infty} \ln a_n = \lim_{n\to\infty} \frac{\ln(1+\frac{1}{n})}{1/n} \underset{\frac{0}{0}}{\overset{L'H}{=}} \lim_{n\to\infty} \frac{\frac{1}{1+1/n}\cdot -1/n^2}{-1/n^2} = \lim_{n\to\infty} \frac{1}{1+1/n}$$

$$= \frac{1}{1+0}$$

$$= 1$$

$$\therefore \lim_{n\to\infty} a_n = \lim_{n\to\infty} e^{\ln a_n} = e^1 = e$$