Chapter 2: Applications of Integration

- Section 2.1 Areas between Curves
  Given a region defined by various familiar curves set up and evaluate a definite integral that calculates the area of the region.

- Volumes of Revolution

  - Section 2.2 Volumes by Slicing (disks, washers or defined cross-sections)
    * Given the base and a characterization of the cross-sections of an object, find its volume.
    * Given a region R in the xy-plane and an axis or rotation (x- or y-axis), determine the volume using disks or washers.
  - Section 2.3 Volumes of Revolution using Shells
    Given a region R in the xy-plane and an axis or rotation (x- or y-axis), determine the volume using shells.

- Section 2.4 Arc Length of a Curve and Surface Area

  - Given a curve C in the xy-plane, find the arc length of C.
  - Given a curve C in the xy-plane and an axis of rotation (x- or y-axis), find the surface area of the volume of rotation.

- Section 2.5 Physical Applications

  - Work. Including Hooke's Law and pumping fluid out of a container.
  - Mass.

- Section 2.7 A Second Look at Exponential and Logarithmic Functions
  You should know how to integrate and differentiate exponential and logarithmic functions as you did in Calc I.

Chapter 3: Techniques of Integration

- For each technique, you should be able to recognize what integrals are suitable to the technique and know how to implement this technique. **Multiple problems from these Sections always appear on Final Exams.**

  - Section 3.1 Integration by Parts
  - Section 3.2 Trigonometric Integrals
  - Section 3.3 Trigonometric Substitution
  - Section 3.4 Partial Fractions

- Section 3.7 Improper Integrals

  How to recognize and evaluate an improper integral. How to determine if an improper integral converges or diverges.

  Practical Notes:

  - The first step of understanding an improper integral is to rewrite it in terms of a limit.

– You must complete the integration and substitution prior to evaluating the limit.

Chapter 5: Sequences and Series

Multiple problems from this Chapter always appear on Final Exams.

- Section 5.1 Sequences

  1. Understand the difference between a *sequence* and a *series*.
  2. Know how to write the terms of a sequence whether the terms are given via an explicit formula or a recursive one.
  3. Know how to write a formula for a sequence given term-by-term (i.e. look for a pattern and generalize it).

- Infinite Series and Convergence

  – Section 5.2 Infinite Series

    1. Know what is meant by *the sequence of partial sums* of a series and be able to find a few of the terms.
    2. Know that a series converges if and only if its sequence of partial sums converges. (This is the definition of convergence for series.)
    3. Know what a geometric series is and how to determine when it converges and when it diverges. If it converges, know to what it converges.
    4. Know that the **harmonic series** is and that it diverges.
    5. Know how to identify and exploit the properties of *telescoping series*.

  – Section 5.3 The Divergence and Integral Tests

    1. How to use the Divergence Test and know its limits.
    2. How to use the Integral Test.
    3. Know what is meant by a p-**series** and under what conditions a p-series converges and diverges.
    4. Any questions about remainders will be extra credit.

  – Section 5.4 Comparison Tests

    1. Know **how** and when to apply the (direct) comparison test and the limit comparison test.

  – Section 5.5 Alternating Series

    1. Know what is meant by an alternating series.
    2. Know how to apply the Alternating Series Test.
    3. Know how to estimate the remainder of a convergent alternating series when the sum of the series is estimated by a partial sum.
    4. Know that absolute convergence implies convergence and why this fact is useful.

5. Know what you must do to *show* that a series is absolutely convergent.

6. Know what you must do to *show* that a series is conditionally convergent.

– Section 5.6 Ratio and Root Tests

1. Know how to apply the ratio test.
2. Know how to apply the root test.
3. Keep in mind that for the root test, you may need to be somewhat careful about your algebra when taking limits.
4. Know how to work with factorials.

Summary: You need to think about what characteristics of series suggest one test or another.

Cautionary Notes:

– You cannot ever assert a series converges or diverges without a justification.
– The only series for which the justification does *not* require the application of a formal test are geometric series, p-series, and the harmonic series.

Chapter 6: Power Series

- Section 6.1 Power Series and Functions

1. Know what is meant by a power series centered at $x = a$.
2. Know how to find the radius of convergence and the center of convergence.
3. Know the power series representation of $f(x) = \frac{1}{1-x}$ and how to use it to find power series representations of other similar functions.

- Section 6.2 Properties of Power Series

1. Know how to operate on power series term-by-term within their radius of convergence including addition, multiplication, differentiation and integration.

- Section 6.3 Taylor and Maclaurin Series

1. Know how to find the Taylor Series of a function $f(x)$ centered at $x = a$:

$$c_n = \frac{f^{(n)}(a)}{n!} \qquad \text{in} \qquad f(x) = \sum_{n=0}^{\infty} c_n (x - a)^n$$

2. Know how to find the nth Taylor polynomial, p_n centered at $x = a$.
3. You should know that a Maclaurin series is a Taylor series with center $a = 0$.
4. Remainder questions will only appear as extra credit.

- Section 6.4 Working with Taylor Series

FE Review

1. Know how to derive the Maclaurin series for the standard functions:

$$\frac{1}{1-x}, e^x, \sin(x), \cos(x), \ln(1+x), \arctan(x), (1+x)^r.$$

2. Know how to use the methods of integration and differentiation to produce new series (like in Section 6.3).

Chapter 7: Parametric Equations and Polar Coordinates

- Parametric Equations

    - Section 7.1: Parametric Equations

        1. know how to sketch a curve described by parametric equations
        2. know how to eliminate the parameter

    - Section 7.2: Calculus of Parametric Curves

        1. know how to find the find the first and second derivatives of parametric curves.
        2. know how to find area under a parametric curve

- Polar Coordinates

    - Section 7.3: Polar Coordinates

        1. Know how to plot points given in polar coordinates and how to sketch polar graphs.
        2. know how to convert between polar coordinates and rectangular coordinates (for both points and equations)