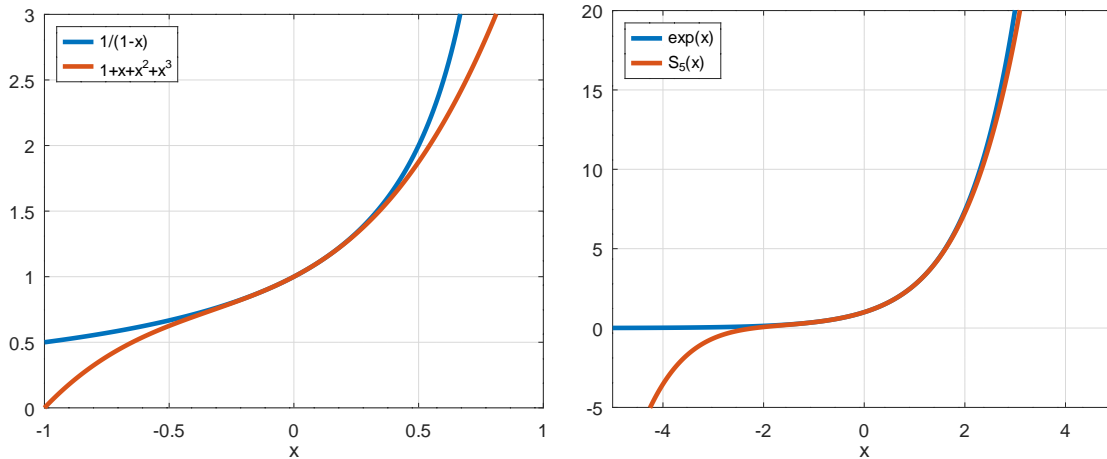# Example of visualizing and multiplying power series

We have two well-known power series:

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \dots$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \dots$$

The figures below show the graphs of the functions compared to a few terms of their power series.

We can multiply series, but only with effort. For example, here is the product of the two series:

$$g(x) = \frac{e^x}{1-x} = \frac{1}{1-x} e^x = \left(1 + x + x^2 + x^3 + \dots\right)\left(1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \dots\right)$$

$$= 1 + 2x + \frac{5}{2}x^2 + \dots$$

Computing the coefficients of the first few terms is like multiplying long polynomials, which is tedious. However, the result is reasonable, and it looks like the figure below.
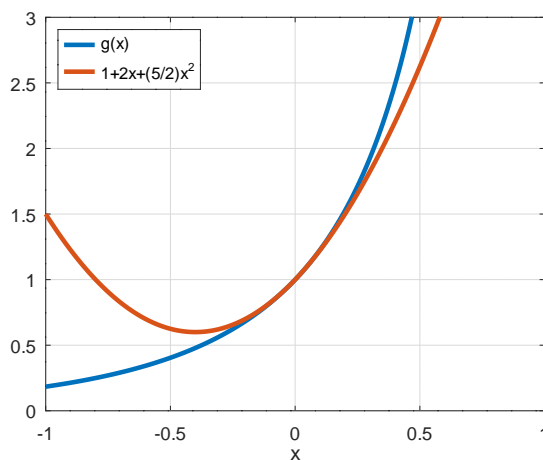
Here is the Matlab code to make the last figure:

```
>> x = -1:.01:1;
>> plot(x,exp(x)./(1-x),x,1+2*x+(5/2)*x.^2)
>> axis([-1 1 0 3]),  grid on,  xlabel x
>> legend('g(x)','1+2x+(5/2)x^2','location','northwest')
```